B-mode spectrum and Inflation models

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by

Pavan R Hebbar

Sandesh S. Kalantre

Anoop Gavankar

Supervisor:

N. Malsawmtluangi

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Chapter 1

CALIBRATING REDSHIFT DISTRIBUTIONS BEYOND SPECTROSCOPIC LIMITS WITH CROSS CORRELATIONS

Cosmic Microwave Background Radiation [1] is the thermal radiation left over from the epoch of recombination in Big Bang cosmology. It was discovered by Penzias and R. W Wilson when they measured an excess antenna temperature of 4.2K. The CMB is a snapshot of the oldest light in our Universe, imprinted on the sky when the Universe was about 380,000 years old. It is the best blackbody source of radiation that we know with a temperature of 2.72548 ± 0.00057 K. Even though the CMB is the most homogeneous source of radiation we know of, it has very small anisotropies.

Inflation is a period of extremely rapid exponential expansion of the universe for a small fraction of a second after the Big Bang, during which the universe expanded in size by a factor of $10^{26} - 10^{27}$. The theory of inflation was proposed to resolve several issues imposed on the standard model of cosmology. Since the first proposal of the theory, several models of inflation have been proposed and studied extensively. Quantum fluctuations during the inflation is believed to have seeded the formation of the large scale structure of the universe. Quantum fluctuations also gave rise to tensor perturbations which sourced the primordial gravitational waves. It is believed that primordial gravitational waves could provide constraint on any model of inflation.
In this report, CMBR spectrum has been analysed to validate different models of inflation.

Overall structure

In this report, Chapter 2 deals with the theory of inflation, Chapter 3 deals with the elements of CMBR spectrum and Chapter 4 deals with the different models that have been analysed.

Chapter 2

Inflation

2.1 The beginning

The standard model of hot Big Bang cosmology has a singularity conventionally taken to be at time t=0. As time t $\rightarrow 0$, the temperature t $\rightarrow \infty$. [2] However, the standard model equations can be safely used only at a temperature comfortably below Planck mass $M_P = 1.22 \times 10^{19} GeV$, as above these temperatures, quantum gravitational effects are expected to become essential. Thus at such a temperature, the universe can be described as a set of initial conditions, and the subsequent conditions are described by the standard model equations.

In the standard model, the initial universe is taken to be completely homogeneous and isotropic, and filled with a gas of effectively massless particles in thermal equilibrium at the initial temperature T_0 .[**?**] Thiiiie universe can be completely described at any subsequent stage subject to the initial conditions, at least in theory. However, as with any theory, a number of mysteries and problems have arisen as a result of the development of the Big Bang theory.

2.2 Problems with the Standard Model

2.2.1 The Horizon Problem

This problem arises from the premise that information travel has a finite upper speed limit. As the universe has a finite age, there is a limit on the size of causally disconnected regions (particle horizons) which is violated by the observed isotropy in the CMBR - for example, wider regions have the same temperature without having the time to communicate that information throughout the region.

2.2.2 The Flatness Problem

The universe may have positive, negative or zero curvature depending on its net energy density(ρ). The energy density of the universe today is very near the critical energy density(ρ_{cr}) corresponding to the zero curvature solution (the borderline between an open and a closed universe). The key point here is that the condition for zero curvature is unstable, and any deviation from critical density will grow with time. Also, the timescale for the amplification of this deviation is of the order of Planck's time. A typical closed universe will reach its maximum size on the order of this timescale, while a typical open universe will have its ρ diminish to values quite less than ρ_{cr}. For the universe to be as it is after roughly 15 billion years since its formation, an extreme fine tuning of initial values of ρ and initial expansion rate (Hubble constant H) is required, so that ρ remains very close to ρ_{cr}. For initial conditions taken at $T_o = 10^{17} GeV$, H_o must be fine tuned to an accuracy of one part in 10^{55}. This incredibly precise initial relationship must be assumed in the Standard Model without explanation.

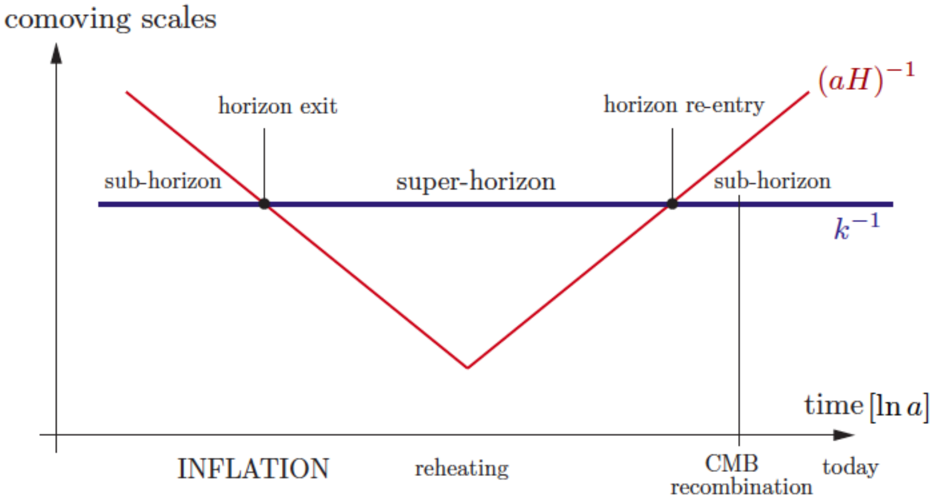

Figure 2.1: Solution to Horizon Problem

2.2.3 Magnetic Monopoles

In the late 1970s, Grand unified theories predicted topological defects in space that would manifest as magnetic monopoles. These objects would be produced efficiently in the hot early universe, resulting in a density much higher than is consistent with observations, given that no monopoles have been found.

2.3 Need For inflation

A resolution to the Horizon Problem is offered by inflationary theory in which a homogeneous and isotropic scalar energy field dominates the universe at some very early period. During inflation, the universe undergoes exponential expansion, and the particle horizon expands much more rapidly than previously assumed, so that regions presently on opposite sides of the observable universe are well inside each other's particle horizon. The observed isotropy of the CMB then follows from the fact that this larger region was in causal contact before the beginning of inflation. The magnetic monopole problem is also resolved, as inflation removes all point defects from the observable universe, in the same way that it removes most of the anisotropy, thus driving the universe extremely close to flatness.

2.4 Slow Roll conditions for inflation

Consider a scalar field ϕ, the *inflaton*, with potential $V(\phi)$.
The inflaton $\phi(t)$ is governed by the Klein-Gordon equation, [3]

$$\ddot{\phi} + 3H\dot{\phi} = -V' \tag{2.1}$$

where H is given by the Friedmann equation,

$$H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V\right] \tag{2.2}$$

From the above two equations, we get the continuity equation ..

$$\dot{H} = -\frac{1}{2}\frac{\dot{\phi}^2}{M_{pl}^2} \tag{2.3}$$

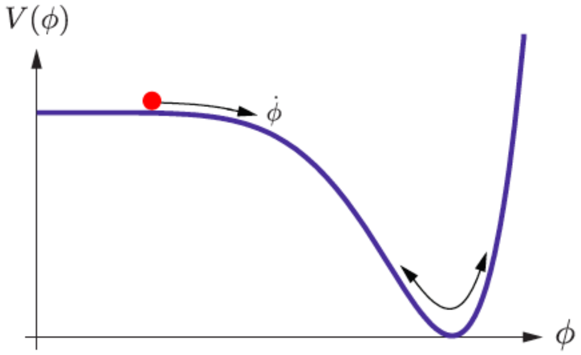

Figure 2.2: Slow Roll Inflation

The slow roll parameters ϵ and η are defined as follows.

$$\epsilon(\phi) = \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2, \eta(\phi) = \frac{{M_{pl}}^2}{8\pi}\frac{V''}{V} \tag{2.4}$$

Using the continuity equation we get,

$$\epsilon = \frac{1}{2}\frac{\dot{\phi}^2}{M_P^2 H^2} \tag{2.5}$$

Inflation occurs only if $\epsilon < 1$, i.e. if potential energy V dominates over kinetic energy $\frac{1}{2}\dot{\phi}^2$. For this condition to persist, the scalar field acceleration must be small. We note that the condition $\frac{1}{2}\dot{\phi}^2 \ll V$ implies that $\epsilon \ll 1$. This condition is called the *slow roll approximation* and is used to simplify the equations of motion. Hence the Friedmann eq. simplifies to

$$H^2 \approx \frac{V}{3M_P^2} \tag{2.6}$$

The Klein-Gordon equation reduces to

$$3H\dot{\phi} \approx -V' \tag{2.7}$$

The conditions for slow-roll inflation to occur are,

$$\epsilon_v \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 and \mid \eta_v \mid \equiv {M_{pl}}^2\frac{V''}{V} \ll 1 \tag{2.8}$$

It is worth noting that though the above conditions are necessary for slow roll inflation to occur, they are not self-sufficient. It is still possible that the scalar field has a very large initial velocity capable enough to override the flatness of the potential V.

Chapter 3

CMB Temparature and Polarisation Anisotropies

Cosmic Microwave Background Radiation [1] is the thermal radiation left over from the epoch of recombination in Big Bang cosmology. Ever since its discovery in 1965, it has been used as an active probe of the early universe. Even though the CMB is the most homogeneous source of radiation we know of, it has very small anisotropies. The largest contributor to these anaisotropies is the dipole contribution of the earth's velocity through space. This dipole contribution is not of cosmological interest. In this chapter, we study the anisotropies (after removing this dipole term) in detail.

3.1 CMB Observables

The CMBR is described by its intensity distribution.[4] Since the spectrum of CMB brightness is close to thermal, in most cases the intensity is described by the temperature $T(\mathbf{n})$ where $\mathbf{n}$ is the direction of observation. The fluctuations represented by $\Delta T(\mathbf{n})$ are of the order of 10^{-5}T, after removing the dipole contribution. These CMB anisotropies $\Delta T(\mathbf{n})/T = \Theta(\mathbf{n})$ at the observaer can be expanded in spherical harmonics.

$$\Theta(\mathbf{n}) \equiv \frac{\Delta T}{T}(\theta\ \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi) \tag{3.1}$$

with $a^*_{lm} = (-1)^m a_{l-m}$ since the temperature is a real quantity.

Assuming Θ to be Gaussian random field we have,

$$\langle a_{lm} a_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'} \tag{3.2}$$

Because of statistical isotropy, the power spectrum is independent of m. Theorectical predictions of CMBR anisotropy are then comapred with observations by computing the C_l's or the correlation function $C(\alpha) = \langle \Theta(\mathbf{n}) \Theta^*(\mathbf{m})$, where $\cos\alpha = \mathbf{n} \cdot \mathbf{m}$

$$C(\alpha) = \sum_{lm} \sum_{l'm'} \langle a_{lm} a^*_{l'm'} \rangle Y_{lm} Y^*_{lm} = \sum_l C_l \frac{2l+1}{4\pi} P_l(cos\alpha) \tag{3.3}$$

The mean-square temperature anisotropy,

$$\langle (\Delta T)^2 \rangle = T^2 C(0) = T^2 \sum_l C_l \frac{2l+1}{4\pi} P_l(cos\alpha) \approx T^2 \int \frac{l(l+1)C_l}{2\pi} dlnl \tag{3.4}$$

with the last approximate equality valid for large l, and so $l(l+1)C_l/2\pi$ is a measure of the power in the temperature anisotropies, per logarithmic interval in l space.

Note that CMB brightness and hence Θ is a function of spacetime location (x, η) where x and η are the conformal spatial and time coordinates respectively. $C(\alpha)$ is calculated by the ensemble average

$\langle\Theta(\mathbf{x_0},\eta_{\mathbf{0}},\mathbf{n})\Theta(\mathbf{x_0},\eta_{\mathbf{0}},\mathbf{m})\rangle$. The Fourier component to Θ, for every $\mathbf{k}$ mode depends on $\mathbf{n}$ only through $\hat{\mathbf{k}}\cdot\mathbf{n}=\mu$ where $\hat{\mathbf{k}}=\mathbf{k}/\mid\mathbf{k}\mid$. Expanding Θ by Fourier Legandre series,

$$\Theta(\mathbf{x_0},\eta_0,\mathbf{n})=\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot\mathbf{x_0}}\sum_l(-i)^l(2l+1)a_l(\hat{\mathbf{k}},\eta_{\mathbf{0}})P_l(\hat{\mathbf{k}}\cdot\mathbf{n}) \tag{3.5}$$

For a homogeneous, isotropic, Gaussian random Θ field,
$\langle a_l(\mathbf{k},\eta_{\mathbf{0}})a^*_{l'}(\mathbf{p},\eta_0)\rangle=\langle\mid a_l(k,\eta_0)\mid^2\rangle\delta_{l,l'}(2\pi)^3\delta^3(\mathbf{k}-\mathbf{p})$ where the power spectrum $\langle\mid a_l(k,\eta_0)\mid^2\rangle$ depends only on $k=\mid\mathbf{k}\mid$. Therefore

$$C(\alpha)=\sum_l\frac{2}{\pi}\int\frac{dk}{k}k^3\langle\mid a_l(k,\eta_0)\mid^2\rangle\frac{2l+1}{4\pi}P_l(cos\alpha) \tag{3.6}$$

Comparing with equation 3.3, we see that,

$$C_l=\frac{2}{\pi}\int\frac{dk}{k}k^3\langle\mid a_l(k,\eta_0)\mid^2\rangle \tag{3.7}$$

3.2 Sources of CMB anisotropies

The major sources of CMB anisotropies have been discussed in this section.[5]

3.2.1 Saches-Wolfe Effect

This is the simplest source of density fluctuations through gravitational redshift. A photon coming from a slightly overdense region is more redshifted than the photon coming from an underdense region. Thus the CMB temperature anisotropies can by calculated due to slightly varying Newtonian potential Φ from the density fluctuations at surface of last scattering

$$\left(\frac{\Delta T}{T}\right)=\frac{1}{3}\Phi \tag{3.8}$$

Fluctuations on large angular scales (low multipoles) are larger than the horizon at the time of last scattering, so this phenomenon is dominant on large angular scales.

3.2.2 Acoustic Oscillations

Matter tends to collapse due to gravity onto regions of higher density. But since baryons and photons are strongly coupled, the photons tend to resist the collapse and push the baryons outside. The gas heats as it compressed and cools as it expands, and this creates fluctuations in the CMB. These oscillations can be used to calculate the curvature of the universe.

3.3 Power Spectrum

The Power spectrum $P(k)$ is defined as the variance per unit logarithmic interval. For convenience the different components of the CMBR spectrum (scalar and tensor) are seperated and denoted by $P_S(k)$ and $P_T(k)$ respectively.

The power law for the scalar spectrum is of form

$$P_S(k)\propto k^{n_S-1} \tag{3.9}$$

where n_S is the scalar spectral index. Similarly the tensor spectrum is given by

$$P_T(k)\propto k^{n_T-1} \tag{3.10}$$

where n_T is the tensor spectral index.

It can be show that for a slow role inflation:

$$\begin{aligned} n_S &= 1 + 2\eta - 6\epsilon \\ n_T &= -2\epsilon \end{aligned} \tag{3.11}$$

For convenience we can calculate $P_S(k)$ and $P_T(k)$ at a pivot value of k and then from the power law we can find the power spectrum at other modes
For scalar power spectrum the pivot value is taken as $k_{S*} = 0.05$.
For tensor power spectrum the pivot value is taken as $k_{T*} = 0.002$.
For these pivot values

$$P_S = \frac{2V}{3\pi^2 M_{pl}^2} \tag{3.12}$$

$$P_T = \frac{1}{12\pi^2 M_{pl}^2} \frac{V^3}{V'^2} \tag{3.13}$$

3.4 Polarisation

In addition to anisotropies in the CMB temperature, we expect the CMB to become polarized via Thomson scattering of the photons from free electrons just before decoupling(see Figure 3.1

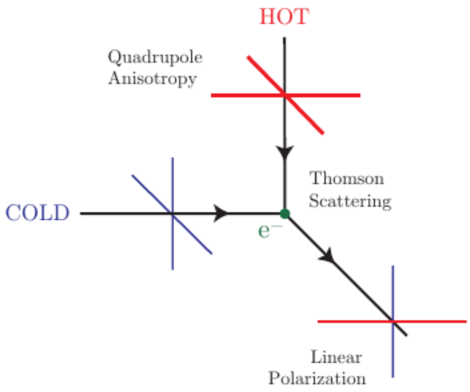

Figure 3.1: Quadrupole anisotropy leading to polarisation

The advantages of studying polarisation in addition to studying the temperature anisotropies are:

1. The detection of polarisation in CMB were important confirmations of gravitational instability paradigm.
2. Polarization spectrum provides the information complementary to temperature spectrum which can be used to break degeneracies and fix cosmological parameters more accurately.
3. Confirms acoustic interpretation of temperature peaks
4. Not affected by physical processes. Hence it is a direct probe to the last scattering surface.
5. We can distinguish the components of the power spectrum and so study the cosmological model more accurately

3.4.1 Stokes parameters

The anisotropy is characterized by intensity matrix tensor $I_{ij}(\hat{n})$. It is a function of the direction on the sky and the two directions perpendicular to it which are used to define its components (,). The Stoke parameters are defined as

$$Q = \frac{I_{11} - I_{22}}{4} \tag{3.14}$$

$$U = \frac{I_{12}}{4} \tag{3.15}$$

and the temperature anisotropy is

$$T = \frac{I_{11} + I_{22}}{4} \tag{3.16}$$

The polarisation magnitude and angle are given by:

$$P = \sqrt{Q^2 + U^2} \tag{3.17}$$

$$\alpha = \frac{1}{2} tan^{-1} \left(\frac{U}{Q} \right) \tag{3.18}$$

The quantity T is invariant under rotation and and should be expanded in terms of scalar spherical harmonics as in equation 3.1.
The quantities Q and U transform under rotation by angle ψ as a spin -2 field $(Q \pm iU)(\hat{n}) \rightarrow e^{\mp 2i\psi}(Q \pm U)(\hat{n})$. The harmonic analysis of $Q \pm U$ can be done by its expansion in terms of tensor(spin -2) spherical harmonics

$$(Q \pm iU)(\hat{n}) = \sum_{lm} a_{\pm 2,lm \pm 2} Y_{lm}(\hat{n}) \tag{3.19}$$

3.4.2 E/B decomposition

The spin-1 polarization field can be decomposed spin-0 quantities, the so-called E- and B-modes i.e instead of moments $a_{\pm 2,lm}$ it is convenient to introduce liner combinations

$$a_{E,lm} \equiv -\frac{1}{2}(a_{2,lm} + a_{-2,lm}) \qquad a_{B,lm} \equiv -\frac{1}{2i}(a_{2,lm} - a_{-2,lm}) \tag{3.20}$$

$$E(\hat{n}) = \sum_{lm} a_{E,lm} Y_{lm}(\hat{n}) \qquad B(\hat{n}) = \sum_{lm} a_{B,lm} Y_{lm}(\hat{n}) \tag{3.21}$$

The angular spectrum are defined as:

$$C_l^{XY} = \frac{1}{2l+1} \sum_m \langle a^*_{X,lm} a_{Y,lm} \rangle, \qquad X, Y = T, E, B, \tag{3.22}$$

The scalar quantities E and B completely specify the linear polarization field. E-mode polarization is curl-free with polarization vectors that are radial around cold spots and tangential around hot spots on the sky. In contrast, B-mode polarization is divergence-free but has a curl: its polarization vectors have vorticity around any given point on the sky.

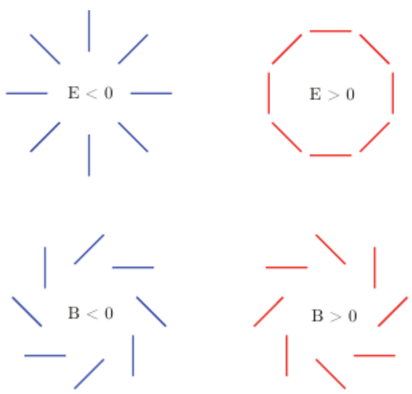

Figure 3.2: E modes and B Modes

Chapter 4

Analysis of Inflation Models

4.1 Models of Inflation

An inflation model can be described by specifying a potential $V(\phi)$. For the purposes of this report, we have studied four different inflation models and used them to derive the Scalar mode (P_S)and Tensor mode fluxes (P_T) and their ratio r under the slow roll approximation.

4.2 Steps of Analysis

1. For a given $V(\phi)$, the under the slow-roll approximation we calculate the power spectrum for scalar and tensor modes, the ratio r and the scalar and tensor spectral indices using the methods described in Chapters 2 and 3.
2. The e-folding number was assumed to be $N = 60$ for all models described.
3. Using the parameters so obtained, the CAMB code was used to obtained the BB mode spectrum.[6]
4. The data from the Planck - BICEP experiment for BB model was compared to the data generated in the previous step to test the feasibility of the model.

4.2.1 Large Field Inflation

$$V(\phi) = M^4 \left(\frac{\phi}{M_P}\right)^2$$

For the purpose of analysis, we have used $M \approx M_P \times 3 \times 10^{-3}$.
This is an example of chaotic inflation model. The field rolls down a quadratic potential and oscillates around the true vacuum position.

Using the slow-roll approximation outlined in Chapter 3, we arrive the following results for the given model:

$$P_S = 1.0013 \times 10^{-8}$$
$$P_T = 1.3241 \times 10^{-9}$$
$$r = 0.1322$$

The data obtained from CAMB was compared with Planck-BICEP data. We see that data generated for this model fits quite well for the given model.

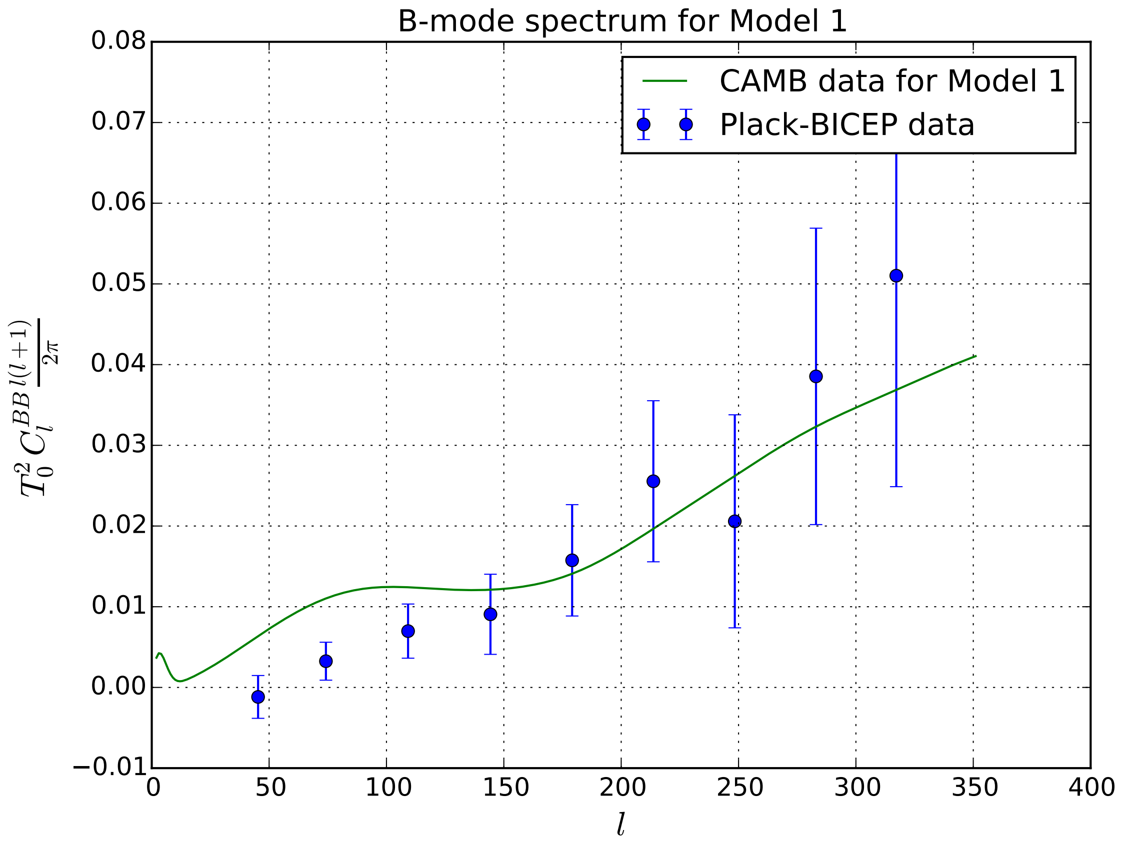

Figure 4.1: Model 1

4.2.2 Mixed Large Field Inflation

$$V(\phi) = M^4 \frac{\phi^2}{M_P^2} \left(1 + \alpha \frac{\phi^2}{M_P^2}\right)$$

The results for this model are arrived to be as follows:

$$P_S = 2.562 \times 10^{-9}$$
$$P_T = 7.0334 \times 10^{-10}$$
$$r = 0.2745$$

4.2.3 Mutated Hilltop Inflation

$$V(\phi) = M^4 \left(1 - sech \frac{\phi}{\mu}\right)$$
$$\frac{\mu}{M_P} \approx 10^{-2}$$
$$\frac{M}{M_P} \approx 10^{-4}$$

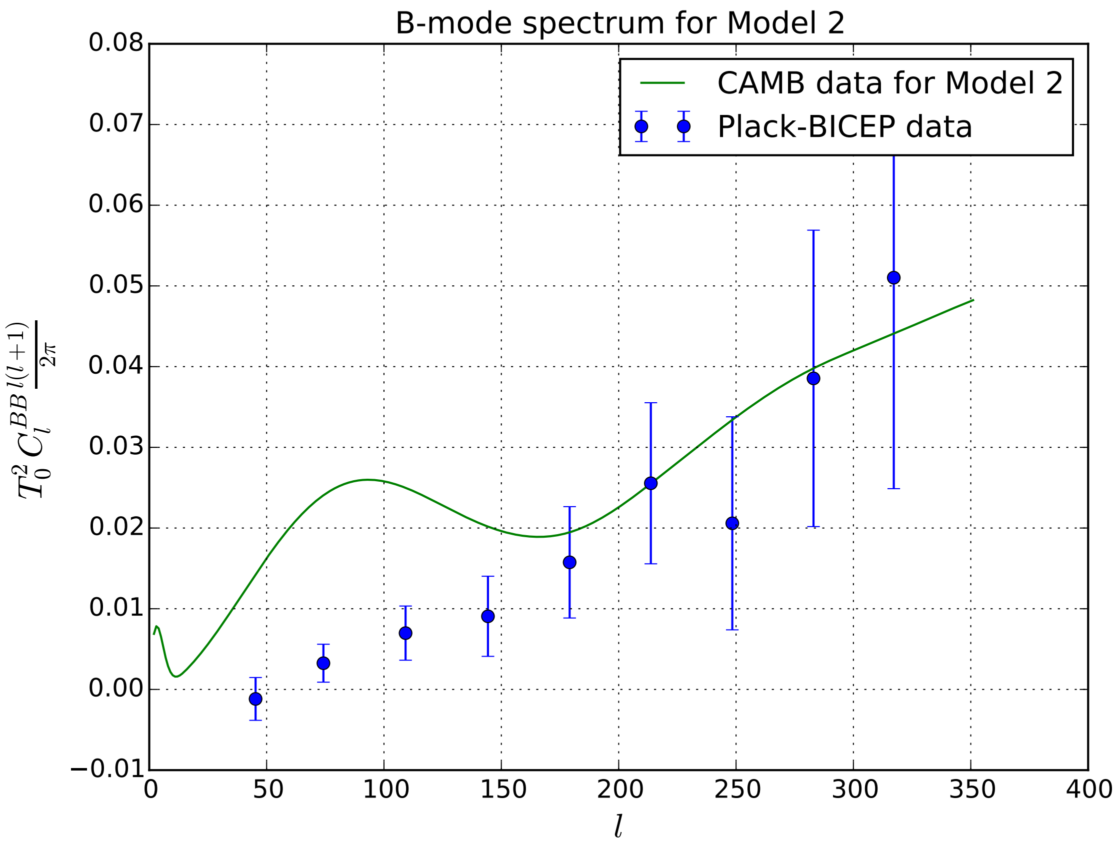

Figure 4.2: Model 2

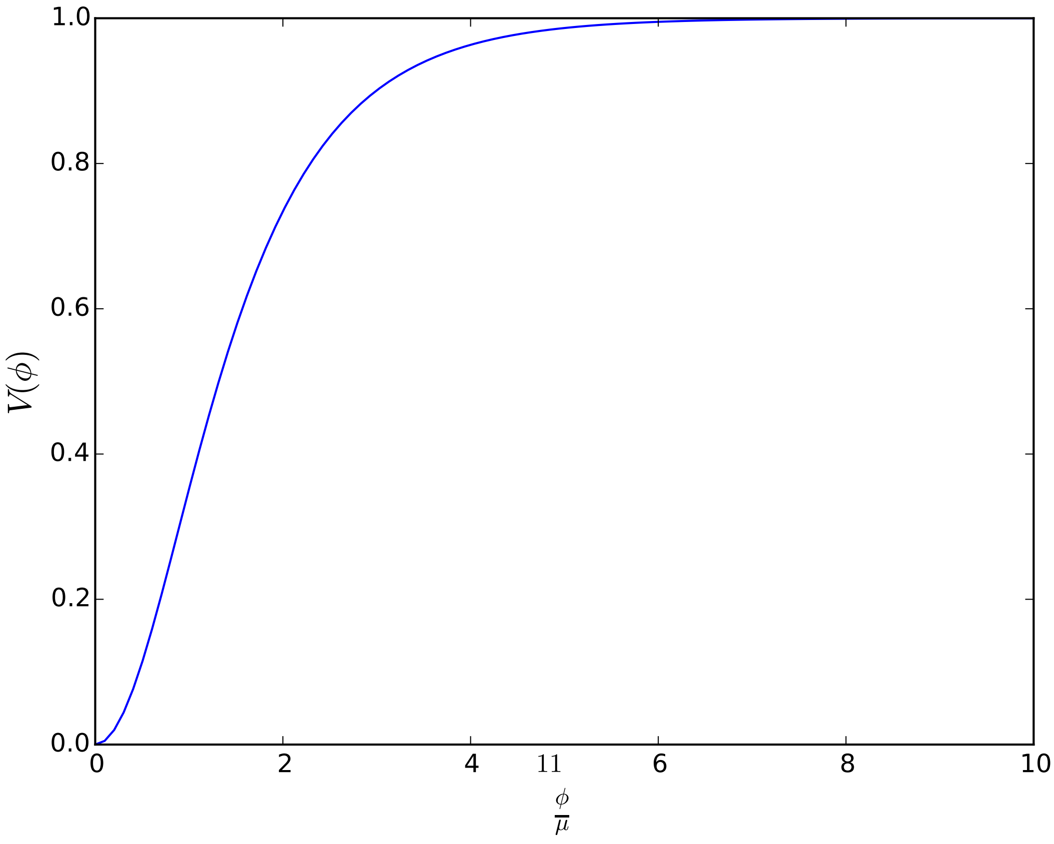

Figure 4.3: Model 3 : Potential

This potential graphically looks like a quadratic potential near the origin.

$$P_S = 3.601 \times 10^{-9}$$
$$P_T = 6.755 \times 10^{-18}$$
$$r = 1.876 \times 10^{-9}$$

We see that similar to the Large Field Inflation model, this model again fits quite well with the observations.

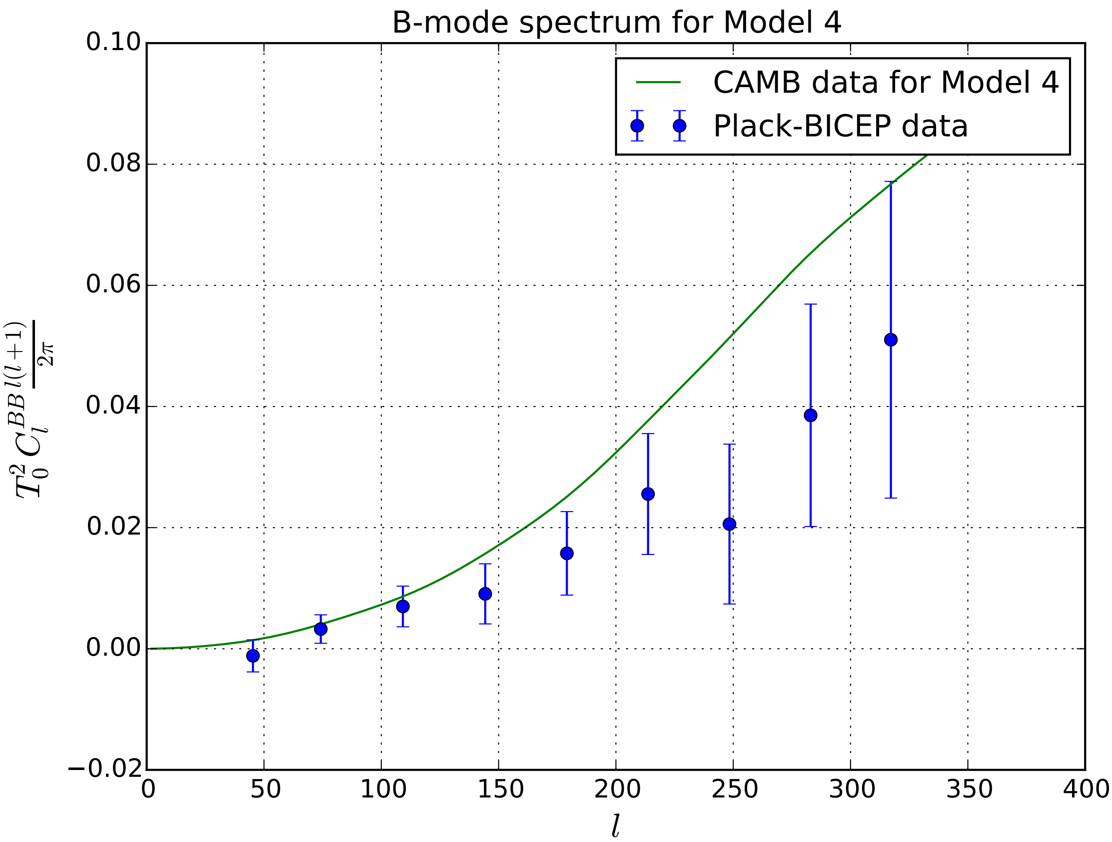

Figure 4.4: Model 3

Chapter 5

Conclusion

We see that the inflation models studied in this report adhere to Planck-BICEP observations.
We see that model 1 fits the best wherein r is of the order of 0.1.
The knowledge of potential of the inflation field can be used to study the origin of the field itself and can potentially lead to a clearer understanding of physics of the early universe.

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