Solving Brio-Wu Shock Tube problem using Godunov Schemes

Supervised Learning Project Presentation

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Presentation Outline

What is Magnetohydrodynamics

- Study of electrically conducting fluids and their flow properties
- Combination of fluid mechanics and electromagnetism.
  Fluid Mech - Navier Stokes
  Electromagnetism - Maxwell's equations
- Primarily used to study plasma flow properties

What is Plasma

- Electrically conducting fluids that are neutral on macroscopic scale
- The number of electrons inside the Debye sphere is large
- Characteristic length scales should be much larger than the Debye length
- Average time between electron-neutral particle collisions be much larger than characteristic time scales of plasma flow.

Presentation Outline

Magnetohydrodynamic equations

- Mass Conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

- Momentum Conservation

$$\frac{\partial (\rho \mathbf{u})}{\partial t} = \rho_e \mathbf{E} + \mathbf{J} \times \mathbf{B} - \nabla \cdot (pI + \frac{1}{2}\mathbf{uu}) + \psi \tag{2}$$

- Energy Conservation

$$\frac{1}{2}\rho \frac{Du^2}{Dt} + \rho \frac{De}{Dt} = -p\nabla \cdot \mathbf{u} + \mathbf{E}.\mathbf{J} + \phi \tag{3}$$

Maxwell's equations

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0 \tag{4}$$

$$\frac{\partial \mathbf{D}}{\partial t} - \nabla \times \mathbf{H} + \mathbf{J} = 0 \tag{5}$$

$$\nabla \cdot \mathbf{D} = \rho_e \tag{6}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{7}$$

Supplemented by the equations

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{J} = 0 \tag{8}$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{9}$$

Ideal MHD Assumptions

- Displacement Current is neglected in comparison to conduction current i.e

$$\frac{\epsilon(\partial \mathbf{E}/\partial t)_{max}}{\sigma \mathbf{E}_{max}} = \frac{\epsilon\omega}{\sigma} = 10^{-13}\omega \tag{10}$$

- Free charge density (ρ_e) is assumed to be zero. Thus-
  - Convection current is negligible in comparison to Conduction current

$$\frac{\rho_e \mathbf{u}}{\sigma E} \cong \frac{(\epsilon E/L)U}{\sigma \mathbf{E}} = \frac{\epsilon U}{\sigma L} \cong 10^{-8} \tag{11}$$

  - Electrostatic forces much smaller than magnetic force

$$\frac{\rho_e \mathbf{E}}{\mathbf{J} \times \mathbf{B}} \cong \frac{\epsilon E^2}{\sigma LVB^2} \cong \frac{\epsilon V^2 B^2}{\sigma LVB^2} = \frac{\epsilon V}{\sigma L} \cong 10^{-8} \tag{12}$$

- Perfect electric conductor

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} \tag{13}$$

Ideal MHD Equations

$$\rho_t + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{14}$$

$$(\rho \mathbf{u})_t + \nabla \cdot \left(\rho \mathbf{u}\mathbf{u} + P^* I - \frac{\mathbf{B}\mathbf{B}}{\mu} \right) = 0 \tag{15}$$

$$\mathbf{B}_t + \nabla \cdot (\mathbf{u}\mathbf{B} - \mathbf{B}\mathbf{u}) = 0 \tag{16}$$

$$E_t + \nabla \cdot \left[(E + P^*)\mathbf{u} - \frac{1}{\mu}(\mathbf{u} \cdot \mathbf{B})\mathbf{B} \right] = 0 \tag{17}$$

Presentation Outline

Assuming that the variations in states are small from equilibrium value

$$\rho'_t + \rho_0 \nabla \cdot \mathbf{u}' = 0 \tag{18}$$

$$\rho_0 \mathbf{u}'_t + a^2 \nabla \rho' - \frac{(\nabla \times \mathbf{b}) \times \mathbf{B}_0}{\mu} = 0 \tag{19}$$

$$\mathbf{b}_t - \nabla \times (\mathbf{v}' \times \mathbf{B}_0) \tag{20}$$

$$\nabla \cdot \mathbf{b} = 0 \tag{21}$$

Note that we have used the approximation of isentropic process. Thus energy equation and momentum equation are the same. These equations yield 3 sets of wave solutions

Alfven Wave

- One set of wave solutions are transverse waves. These contribute to change in fluid velocity and magnetic field while the pressure and density do not vary
- Effect of external magnetic field is a combination of an isotropic pressure of $B^2/2\mu$ and a tension of B^2/μ
- Wave propagation possible through this magnetic tension

$$\mathbf{A} = \frac{\mathbf{B}}{\sqrt{\mu\rho}} \tag{22}$$

Slow and Fast MHD waves

- The other two sets of wave solutions are longitudnal in nature. These are collectively called as magneto-sonic or magneto-acoustic waves.
- Wave nature depends on direction of wave propagation
- If wave is propagating in the direction of magnetic field, then they behave like sound waves ($a = \sqrt{\gamma p/\rho}$)
- Perpendicular to magnetic field, wave propagation also involves the compression and rarefaction of magnetic field lines along with pressure and density

$$V = \sqrt{a^2 + A^2} \tag{23}$$

- These two sets of wave solutions are called as Fast and Slow MHD waves

Slow and Fast MHD waves

- System of coupled waves which vary the pressure, density, in-plane components of magnetic field etc.

$$c_{f,s}^2 = \frac{1}{2}[(a^2 + A^2)] \pm \sqrt{(a^2 + A^2)^2 - 4a^2 A^2 cos^2\theta]} \quad (24)$$

- If Alfven wave is greater that the speed of sound, then , parallel to **B**, the fast wave combines with the transverse (alfven) wave and the slow wave behaves as a pure sound wave.
- If Alfven wave is lesser that the speed of sound, then , parallel to **B**, the slow wave combines with the transverse (alfven) wave and the fast wave behaves as a pure sound wave.
- Perpendicular to the magnetic field, only the fast MHD wave exists with propagation speed given in 23

Presentation Outline

Magnetohydrodynamic Discontinuity

- Smooth but rapid transitions through a region much smaller than the overall dimensions of interest.
- Equations below analysed from frame of shock
- For a ideal plasma(i.e perfect conductor and no excess charge and displacement current)

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} \tag{25}$$

- Now from Poisson's equation $\nabla \cdot \mathbf{E} = 0$ and the fact that $\nabla \times \mathbf{E} = 0$,

$$\nabla \cdot (\mathbf{u} \times \mathbf{B}) = \nabla \times (\mathbf{u} \times \mathbf{B}) = 0 \tag{26}$$

- Thus the vector $(\mathbf{v} \times \mathbf{B})$ doesn't change across a discontinuity

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} + \frac{\partial \mathbf{g}}{\partial y} + \frac{\partial \mathbf{h}}{\partial z} \tag{27}$$

$$\mathbf{u} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ B_x \\ B_y \\ B_z \\ E \end{pmatrix}, \mathbf{f} = \begin{pmatrix} \rho u \\ \rho u^2 + P^* \\ \rho uv - B_y B_x \\ \rho uw - B_z B_x \\ 0 \\ uB_y - vB_x \\ uB_z - wB_x \\ (E + P^*)u - B_x(uB_x + vB_y + wB_z) \end{pmatrix}$$

Computational MHD

$$\mathbf{g} = \begin{pmatrix} \rho v \\ \rho vu - B_x B_y \\ \rho v^2 + P^* \\ \rho vw - B_z B_y \\ vB_x - vB_y \\ 0 \\ vB_z - wB_x \\ (E + P^*)v - B_y(uB_x + vB_y + wB_z) \end{pmatrix}$$

$$\mathbf{h} = \begin{pmatrix} \rho w \\ \rho wu - B_x B_z \\ \rho wv - B_z B_y \\ \rho w^2 + P^* \\ wB_x - uB_z \\ wB_y - vB_z \\ 0 \\ (E + P^*)w - B_z(uB_x + vB_y + wB_z) \end{pmatrix}$$

Presentation Outline

1D MHD

- Gradients are assumed to exist only along the x-direction

$$\frac{\partial b_x}{\partial x} = 0 \qquad (28)$$

$$\frac{\partial b_x}{\partial t} = 0 \qquad (29)$$

$$\frac{\partial b_y}{\partial t} = B_{0_x}\frac{\partial u'_y}{\partial x} - B_{0_y}\frac{\partial u'_x}{\partial x} \qquad (30)$$

$$\frac{\partial b_z}{\partial t} = B_{0_x}\frac{\partial u'_z}{\partial x} \qquad (31)$$

$$\frac{\partial \rho'}{\partial t} = -\rho_0\frac{\partial u'_x}{\partial x} \qquad (32)$$

$$\frac{\partial u'_x}{\partial t} = -\frac{a^2}{\rho_0}\frac{\partial \rho'}{\partial x} - \frac{1}{\rho_0\mu}B_{0_y}\frac{\partial b_y}{\partial x} \qquad (33)$$

$$\frac{\partial u'_y}{\partial t} = \frac{1}{\rho\mu}B_{0_x}\frac{\partial b_y}{\partial x} \qquad (34)$$

$$\frac{\partial u'_z}{\partial t} = \frac{1}{\rho\mu}B_{0_x}\frac{\partial b_z}{\partial x} \qquad (35)$$

Alfven wave

- From equations 28 and 29, we see that $b_x = constant$
- Differentiate equation 31 w.r.t t and substitute equation 35. We get

$$\frac{\partial^2 b_z}{\partial t^2} = A_x^2 \frac{\partial^2 b_z}{\partial x^2} \tag{36}$$

where $A_x = \frac{B_{0_x}}{\sqrt{\rho_0 \mu}}$

- Similarly differentiate 35 w.r.t t and substitute using equation 31

$$\frac{\partial^2 u_z}{\partial t^2} = A_x^2 \frac{\partial^2 u_z}{\partial x^2} \tag{37}$$

Magneto-sonic waves

SLP Presentation

- Differentiate equation30 w.r.t t and substitute v'_y from equation 34 and v'_x from equation 33. We get

$$\frac{\partial^2 b_y}{\partial t^2} = \frac{B_0^2}{\rho_0 \mu} \frac{\partial^2 b_y}{\partial x^2} + \frac{a^2}{\rho_0} B_{0_y} \frac{\partial^2 \rho'}{\partial x^2} \tag{38}$$

- Differentiate equation 32 w.r.t t and substitute v'_x from equation 33. We get

$$\frac{\partial^2 \rho'}{\partial t^2} = a^2 \frac{\partial^2 \rho'}{\partial x^2} + \frac{B_{0_y}}{\mu} \frac{\partial^2 b_y}{\partial x^2} \tag{39}$$

- Coupled equations

Magneto-sonic waves

- Substitute $b_y = b_{0_y} e^{i(kx-\omega t)}$ and $\rho' = \rho'_0 e^{i(kx-\omega t)}$. We get

$$\left(c^2 - \frac{B_0^2}{\rho\mu}\right) b_{0_y} - \frac{a^2 B_{0_y}}{\rho_0} \rho'_0 = 0 \tag{40}$$

$$(c^2 - a^2)\rho'_0 - \frac{B_{0_y}}{\mu} b_{0_y} = 0 \tag{41}$$

where $\mathbf{c} = \omega \mathbf{k}/k^2$ or $c = \omega/k$

- Applying the condition for non-trivial solutions,

$$c_{f,s} = \frac{1}{2}\left(\sqrt{a^2 + 2aA_x + A^2} \pm \sqrt{a^2 - 2aA_x + A^2}\right) \tag{42}$$

Presentation Outline

MHD shock relations

From the ideal MHD equations we get:

$$[\rho u] = 0 \quad (43) \qquad \left[\rho u^2 + p + \frac{1}{2\mu}(B_y^2 + B_x^2)\right] = 0 \quad (44)$$

$$\left[\rho u v - \frac{B_x B_y}{\mu}\right] = 0 \quad (45) \qquad \left[\rho u w - \frac{B_x B_z}{\mu}\right] = 0 \quad (46)$$

$$[uB_y - vB_x] = 0 \quad (47) \qquad [wB_x - uB_z] = 0 \quad (48)$$

$$\left[\rho u h_0 + \frac{u}{\mu}(B_y^2 + B_z^2) - \frac{B_x}{\mu}(vB_y + wB_z)\right] = 0 \quad (49)$$

Rankine-Hugoniot Equation

- Rearranging equation 49 using equations 43, 45 and 46 to remove u, v, w terms, we get

$$[h] + \frac{1}{2}m^2[\tau^2] + \frac{1}{\mu}[\tau B_t^2] - \frac{1}{2m^2}\frac{B_x^2}{\mu^2}[B_t^2] = 0 \qquad (50)$$

where $m = \rho u$ and $\tau = 1/\rho$

- From equation 44,

$$m^2 = \frac{p_2 - p_1 + \frac{1}{2\mu}(B_{t_2}^2 - B_{t_1}^2)}{\tau_1 - \tau_2} \qquad (51)$$

- From equations 45, 47 and equations 46, 48,

$$m^2 = \frac{B_x^2}{\mu}\frac{[B_y]}{[\tau B_y]} \qquad (52) \qquad\qquad m^2 = \frac{B_x^2}{\mu}\frac{[B_z]}{[\tau B_z]} \qquad (53)$$

- Combining above equations and using $h = e + p/\rho$

$$e_2 - e_1 + \frac{1}{2}(p_1 + p_2)(\tau_2 - \tau_1) + \frac{1}{4\mu}((B_{z_2} - B_{z_1})^2 + (B_{y_2} - B_{y_1})^2)(\tau_2 - \tau_1) = 0 \qquad (54)$$

Friedrichs' Shock equations

Defining suitable average values, equations 43 to 49 can be written as:

$$m[\tau] - [u] = 0 \quad (55)$$

$$m[u] + [p] + \frac{1}{\mu}(\tilde{B}_y[B_y] + \tilde{B}_z[B_z]) = 0 \quad (56)$$

$$m[v] - \frac{B_x}{\mu}[B_y] = 0 \quad (57)$$

$$m[w] - \frac{B_x}{\mu}[B_z] = 0 \quad (58)$$

$$m\tilde{\tau}[B_y] + \tilde{B}_y[u] - B_x[v] = 0 \quad (59)$$

$$m\tilde{\tau}[B_z] + \tilde{B}_z[u] - B_x[v] = 0 \quad (60)$$

Friedrichs' Shock equations

- Above equations form a system of equations for variables $[u]$, $[v]$, $[w]$, $[B_y]$, $[B_z]$, $[\tau]$
- Solve for the non-triviality of these equations by setting the determinant as zero.
- It gives three solutions of m -

$$m_A = \frac{B_x}{\sqrt{\mu\tilde{\tau}}} \tag{61}$$

$$m_{f,s}^2 = \frac{1}{\sqrt{2}}\left(\frac{\tilde{B}^2}{\mu\tilde{\tau}} - \frac{[p]}{[\tau]} \pm \sqrt{\frac{[p]^2}{[\tau]^2} + \frac{\tilde{B}^2}{\mu^2\tilde{\tau}^2} - \frac{2[p]}{[\tau]\tilde{\tau}}\frac{(\tilde{B_x}^2 + \tilde{B_y}^2 + \tilde{B_z}^2)}{\mu}}\right) \tag{62}$$

Friedrichs' Shock equations

$$[\tau] = -Cm\left(m^2\tilde{\tau} - \frac{B_x^2}{\mu}\right)\left(m^2\tilde{\tau} - \frac{\tilde{B}^2}{\mu}\right) \tag{63}$$

$$[u] = -Cm^2\left(m^2\tilde{\tau} - \frac{B_x^2}{\mu}\right)\left(m^2\tilde{\tau} - \frac{\tilde{B}^2}{\mu}\right) \tag{64}$$

$$[v] = Cm^2\left(\frac{1}{\mu}B_x\tilde{B}_y\right)\left(m^2\tilde{\tau} - \frac{\tilde{B}^2}{\mu}\right) \tag{65}$$

$$[w] = Cm^2\left(\frac{1}{\mu}B_x\tilde{B}_z\right)\left(m^2\tilde{\tau} - \frac{\tilde{B}^2}{\mu}\right) \tag{66}$$

$$[B_y] = Cm^3\tilde{B}_y\left(m^2\tilde{\tau} - \frac{\tilde{B}^2}{\mu}\right) \tag{67}$$

$$[B_z] = Cm^3\tilde{B}_z\left(m^2\tilde{\tau} - \frac{\tilde{B}^2}{\mu}\right) \tag{68}$$

Fast and Slow shocks

- On solving the Friedrichs' Shock equations, one set of equations can be shown to satisfy

$$m^2 = \frac{(-[p]/[\tau])(m^2 - B_x^2/\mu\tilde{\tau})}{m^2 - \tilde{B}^2/\mu\tilde{\tau}} \tag{69}$$

$$m_s^2 < \frac{B_x^2}{\mu\tilde{\tau}} \tag{70}$$

$$m_f^2 > \frac{\tilde{B}^2}{\mu\tilde{\tau}} > \frac{B_x^2}{\mu\tilde{\tau}} \tag{71}$$

- The fast shock velocity of a weak fast shock depends on the orientation of the magnetic field too while that of a weak shock wave depends on the normal component alone

$$[B_t^2] = -\frac{2m^2[\tau]\tilde{B_t}^2}{m^2\tilde{\tau} - B_x^2/\mu} \tag{72}$$

- Thus $|B_t|$ increases across a fast shock and decreases across a slow shock

Fast and Slow Shock

- Switch-off shock - Slow shock for which B_{t_2} is zero but $B_{t_1} \neq 0$
- Switch on shock - Fast shock with $B_{t_2} > 0$ even though $B_{t_1} = 0$
- Two dimensional shocks i.e $B_{t_1} || B_{t_2}$ so we can assume that the $B_{z_1} = B_{z_2}$

Alfven Shocks

- Density doesn't change i.e
  $[\rho] = 0 \implies [e] = 0 \implies [S] = 0$
- Only tangential magnetic field and tangential velocity changes across shock
- From 64, we see that $[B_t^2] = 0$
- Rotation of tangential component of magnetic field in the plane of the shock

$$\rho_1 V_1^2 + p_1 + \frac{B_{y_1}^2}{2\mu} = \rho_1 V_1 u_2 + p_2 + \frac{B_{y_2}^2}{\mu} \tag{73}$$

$$\frac{B_x}{\mu} B_{y_1} = \frac{B_x}{\mu} B_{y_2} - \rho_1 V_1 v_2 \tag{74}$$

$$V_1 B_{y_1} = u_2 B_{y_2} - B_x v_2 \tag{75}$$

$$V_1 \left(\frac{\gamma p_1}{\gamma - 1} + \frac{\rho_1 V_1^2}{2} + \frac{B_{y_1}^2}{\mu} \right) = \frac{\gamma}{\gamma - 1} p_2 u_2 + \frac{\rho_1 V_1}{2}(u_2^2 + v_2^2) + \frac{V_1 B_{y_1}}{\mu} \tag{76}$$

Shocks in perfect gas

- Seven types of shocks
- Alfven Shock: $[B_y^2] = 0$
- Fast Shocks: $[B_y^2] > 0$
    - Type 1: $q \geq 1 - \frac{\gamma}{\gamma - 1} sin^2\theta$
    - Type 2: $q < 1 - \frac{\gamma}{\gamma - 1} sin^2\theta$
- Slow Shocks: $[B_y^2] < 0$
    - Type 1: $q \geq 1 - \gamma sin^2\theta$
    - Type 2: $q < 1 - \gamma sin^2\theta$

where $q = \frac{\gamma p_1 \mu}{B_1^2}$

Presentation Outline

1D computational MHD

- Assuming that the gradients exist only in x-direction

$$\mathbf{u}_t + \mathbf{f}(\mathbf{u}_x) = 0 \tag{77}$$

$$\mathbf{u} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ B_y \\ B_z \\ E \end{pmatrix}, \mathbf{f}(\mathbf{u}) = \begin{pmatrix} \rho u \\ \rho u^2 + P^* \\ \rho uv - B_y B_x \\ \rho uw - B_z B_x \\ 0 \\ uB_y - vB_x \\ uB_z - wB_x \\ (E + P^*)u - B_x(uB_x + vB_y + wB_z) \end{pmatrix}$$

1D MHD Eigenstructure

- $\lambda_e = u$
- $\lambda_a^\pm = u \pm c_a$
- $\lambda_f^\pm = u \pm c_f$
- $\lambda_s^\pm = u \pm c_s$
- Eigenvectors proposed by Jeffrey and Tanuiti, become singular at points where two or more eigenvalue coincide
- Eigenvalues were re-normalised by Brio and Wu to ensure that they are well defined at all points. But they fail at the triple umbilic point
- In this project Brio-Wu eigenvectors have been used

1D MHD Wave structure

- Equations are non convex in structure leading to intermediate shocks
- These non evolutionary discontinuities are solutions of MHD Rankine Hugoniot Jump conditions
- Initially, all non evolutionary shocks were rejected, but they were observed in the Brio-Wu Shock Tube solution and also in Voyager I data
- The exact wave structure is still under debate

Presentation Outline

Godunov schemes for 1D MHD

- Assumes piecewise constant distribution of form

$$\mathbf{U}_i^n = \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \tilde{\mathbf{U}}(x, t^n) dx \tag{78}$$

- Conservative law given by

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t}{\Delta x} \left[\mathbf{F}_{i+1/2}^n - \mathbf{F}_{1-1/2}^n \right] \tag{79}$$

$$\mathbf{F}_{i+1/2}^n = \mathbf{F}(\mathbf{U}_{i+1/2}^n) \tag{80}$$

- $\mathbf{U}_{i+1/2}^n$ estimated by solving local Riemann problem i.e $\mathbf{U}_{1+1/2}^n = RP[\mathbf{U}_i^n, \mathbf{U}_{i+1}^n]$
- Solve two Riemann problems $RP[\mathbf{U}_{i-1}^n, \mathbf{U}_i^n]$, $RP[\mathbf{U}_i^n, \mathbf{U}_{i+1}^n]$ for conservative law above, take integral average for cell i of the combined solutions and assign it to $\mathbf{U}_i^{n+1}$

Linear Riemann Solver

Roe averaging has been used to estimate $\mathbf{u}^n_{i+1/2}$

$$\bar{\rho} = \sqrt{\rho_L \rho_R} \qquad (81) \qquad \bar{\mathbf{u}} = \frac{\sqrt{\rho_L}\mathbf{u}_L + \sqrt{\rho_R}\mathbf{u}_R}{\sqrt{\rho_L} + \sqrt{\rho_R}} \qquad (82)$$

$$\bar{\mathbf{B}} = \frac{\sqrt{\rho_L}\mathbf{B}_L + \sqrt{\rho_R}\mathbf{B}_R}{\sqrt{\rho_L} + \sqrt{\rho_R}} \qquad (83) \qquad \bar{H} = \frac{\sqrt{\rho_L}H_L + \sqrt{\rho_R}H_R}{\sqrt{\rho_L} + \sqrt{\rho_R}} \qquad (84)$$

- Rarefaction waves are assumed to be shocks
- Now evaluate Jacobian at the row avaeraged values
- Diagonalize the Jacobian $\mathbf{\Lambda} = \mathbf{K}^{-1}\mathbf{A}\mathbf{K}$ and consider $\mathbf{V} = \mathbf{K}^{-1}\mathbf{U}$
- Now calculate $\mathbf{V_{i+1/2}}$ using the fact that only one component of $\mathbf{V}$ will change across an eigen value wave
- $\mathbf{U_{i+1/2}} = \mathbf{K}\mathbf{V_{i+1/2}}$

Presentation Outline

Brio-Wu Shock Tube

- Extension of Sod's shock tube problem in MHD
- Anti-parallel magnetic field components on either side of initial discontinuity
- Consists of two fast refractions, two slow shocks and a contact discontinuity

$$\mathbf{w_L} = [1, 0, 0, 0, 1, 0, 1] \tag{85}$$

$$\mathbf{w_R} = [0.125, 0, 0, 0, -1, 0, 1] \tag{86}$$

Presentation Outline

Simulation set up

- Domain of x from $(-1, 1)$
- $\Delta t = 0.2$
- CFL number assumed to be constant $= 0.475$
- Results have been properly validated

Density plot

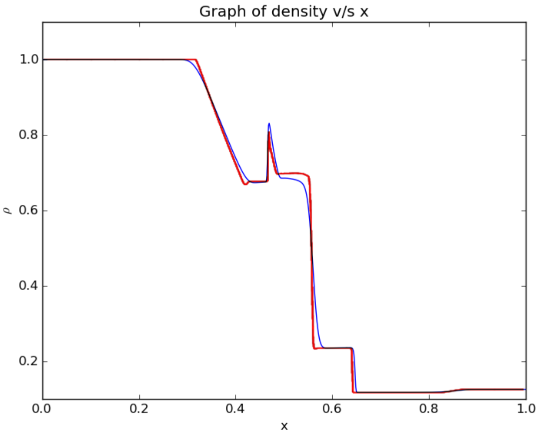

Figure: Density

X-velocity plot

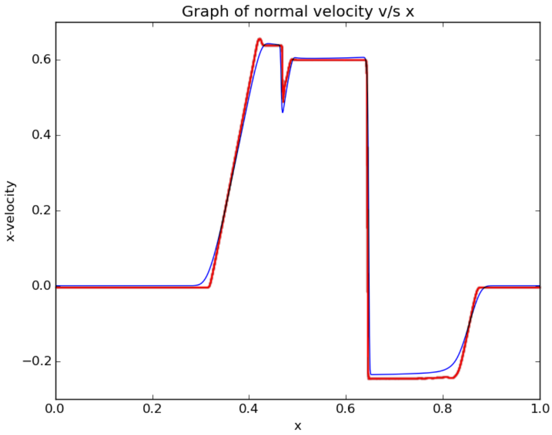

Figure: u_x

Y-velocity plot

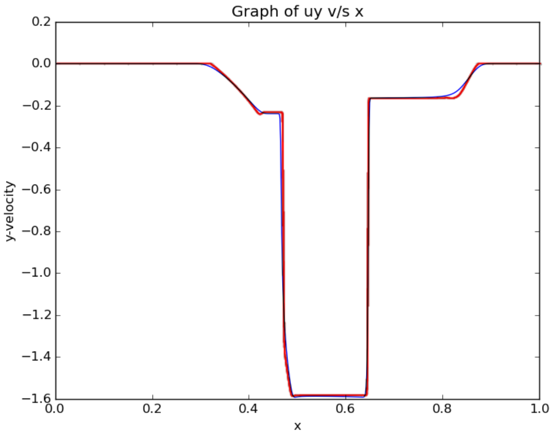

Figure: u_y

Tangential magnetic field

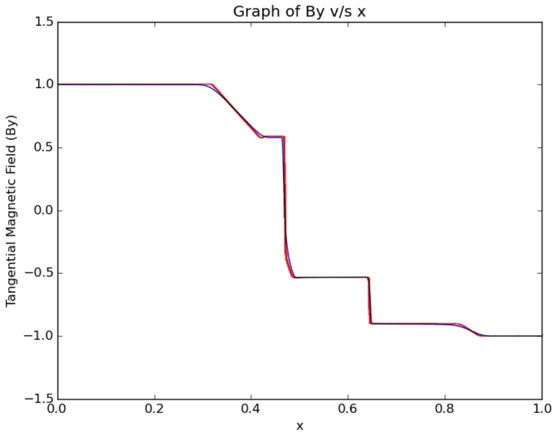

Figure: B_y

Explanations

There are 4 waves formed in the shock tube. From left to right, they can be listed as follows:

- Fast rarefaction waves
- Slow Compound waves
- Contact discontinuity - Not a wave as pressure remains constant
- Slow shock
- Fast rarefaction wave

Conclusion

- The possibility of compound wave/intermediate shocks was shown
- The variation of ρ, $\mathbf{u}$, $\mathbf{B}$, and p were shown to follow the Rankine - Hugonoit Jump Conditions
- The nature of these waves is very different from the normal hydrodynamic case owing to the non convexity of the equations

Presentation Outline

Bibliography